

MATHEMATICS-III

BS-205A, Option II

Time : Three Hours]

[Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit.

Unit I

1. (a) Define the following :

2.5×3=7.5

(i) D'Alembert's ratio test

(ii) Cauchy root test

(iii) Raabe's test.

- (b) Test for convergence of the series :

7.5

$$\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3} \cdot \frac{2}{5}\right)^2 + \left(\frac{1}{3} \cdot \frac{2}{5} \cdot \frac{3}{7}\right)^2 + \dots$$

2. (a) Find the Fourier series of the function

$$f(x) = x + x^2, \text{ in } -\pi < x < \pi. \quad 7.5$$

- (b) Express
- $f(x) = e^x$
- as half-range sine series in

$$0 < x < 1. \quad 7.5$$

Unit II

3. (a) Solve the DE :

7.5

$$(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$$

- (b) Solve :

7.5

$$x - yp = ap^2.$$

4. (a) Solve the ODE :

7.5

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x.$$

- (b) Solve :

7.5

$$x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} - 12y = x^3 \log x.$$

Unit III

5. (a) Evaluate $\iint xy(x+y)dx dy$ over the area between

$$y = x^2 \text{ and } y = x.$$

7.5

- (b) Change order of integration and solve :

7.5

$$\int_0^4 \int_y^4 \frac{x}{x^2 + y^2} dx dy.$$

6. (a) Evaluate :

7.5

$$\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z} dz dy dx.$$

- (b) Find, by triple integration, the volume of the sphere

$$x^2 + y^2 + z^2 = a^2.$$

7.5

Unit IV

7. (a) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ , where Q is the point $(5, 0, 4)$. 7.5

- (b) Prove that :

$$\operatorname{div}(r^n \cdot \vec{r}) = (n+3)\vec{r}^n.$$

8. (a) Evaluate $\iint_S \vec{F} \cdot \hat{n} \cdot dS$, where $\vec{F} = y\hat{i} + 2x\hat{j} - z\hat{k}$, and S is the surface of the plane $2x + y = 6$ in the first octant cut-off by the plane $z = 4$. 7.5



- (b) Evaluate $\oint_C (e^x dx + 2y dy - dz)$; by Stokes' theorem, where C is the curve $x^2 + y^2 = 4, z = 2$. 7.5